

# ALGEBRAIC TECHNIQUES...

# Sequences

@whisto\_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the  $n$ th term
- Recognise geometric sequences and other sequences that arise

## Keywords

**Sequence:** items or numbers put in a pre-decided order

**Term:** a single number or variable

**Position:** the place something is located

**Linear:** the difference between terms increases or decreases (+ or -) by a constant value each time

**Non-linear:** the difference between terms increases or decreases in different amounts, or by  $x$  or  $\div$

**Difference:** the gap between two terms

**Arithmetic:** a sequence where the difference between the terms is constant

**Geometric:** a sequence where each term is found by multiplying the previous one by a fixed non zero number

## Linear and Non Linear Sequences

**Linear Sequences** – increase by addition or subtraction and the same amount each time

**Non-linear Sequences** – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

**Fibonacci Sequence** – look out for this type of sequence

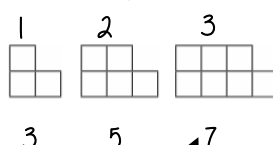
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms



## Sequence in a table and graphically

**Position:** the place in the sequence



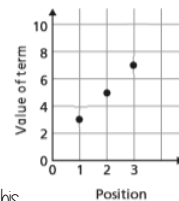
**Term:** the number or variable (the number of squares in each image)

In a table

| Position | 1 | 2 | 3 |
|----------|---|---|---|
| Term     | 3 | 5 | 7 |

+2      +2

**Graphically**



"The term in position 3 has 7 squares"

Because the terms increase by the same addition each time this is **linear** – as seen in the graph

## Sequences from algebraic rules

This is substitution!

$$3n + 7$$

$$3n^2 + 7$$

This will be linear - note the single power of  $n$ . The values increase at a constant rate

This is not linear as there is a power for  $n$

$$2n - 5 \rightarrow$$

Substitute the number of the term you are looking for in place of 'n'

- eg
- 1<sup>st</sup> term =  $2(1) - 5 = -3$
  - 2<sup>nd</sup> term =  $2(2) - 5 = -1$
  - 100<sup>th</sup> term =  $2(100) - 5 = 195$

## Checking for a term in a sequence

Form an equation

Is 201 in the sequence  $3n - 4$ ?

Algebraic rule

$$3n - 4 = 201$$

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

## Complex algebraic rules

Misconceptions and comparisons

$$2n^2$$

$$(2n)^2$$

2 times whatever  $n$  squared is

2 times  $n$  then square the answer

- eg
- 1<sup>st</sup> term =  $2 \times 1^2 = 2$
  - 2<sup>nd</sup> term =  $2 \times 2^2 = 8$
  - 100<sup>th</sup> term =  $2 \times 100^2 = 2000$

- eg
- 1<sup>st</sup> term =  $(2 \times 1)^2 = 4$
  - 2<sup>nd</sup> term =  $(2 \times 2)^2 = 16$
  - 100<sup>th</sup> term =  $(2 \times 100)^2 = 40000$

$$n(n + 5)$$

- eg
- 1<sup>st</sup> term =  $1(1 + 5) = 6$
  - 2<sup>nd</sup> term =  $2(2 + 5) = 14$
  - 100<sup>th</sup> term =  $100(100 + 5) = 10500$

You don't need to expand the expression

## Finding the algebraic rule

This is the 4 times table  $\rightarrow$  4, 8, 12, 16, 20....

$$4n$$

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

$$4n + 3$$